Digital Integrated Circuit Design
CMPE 530/630

Semiconductor Basics

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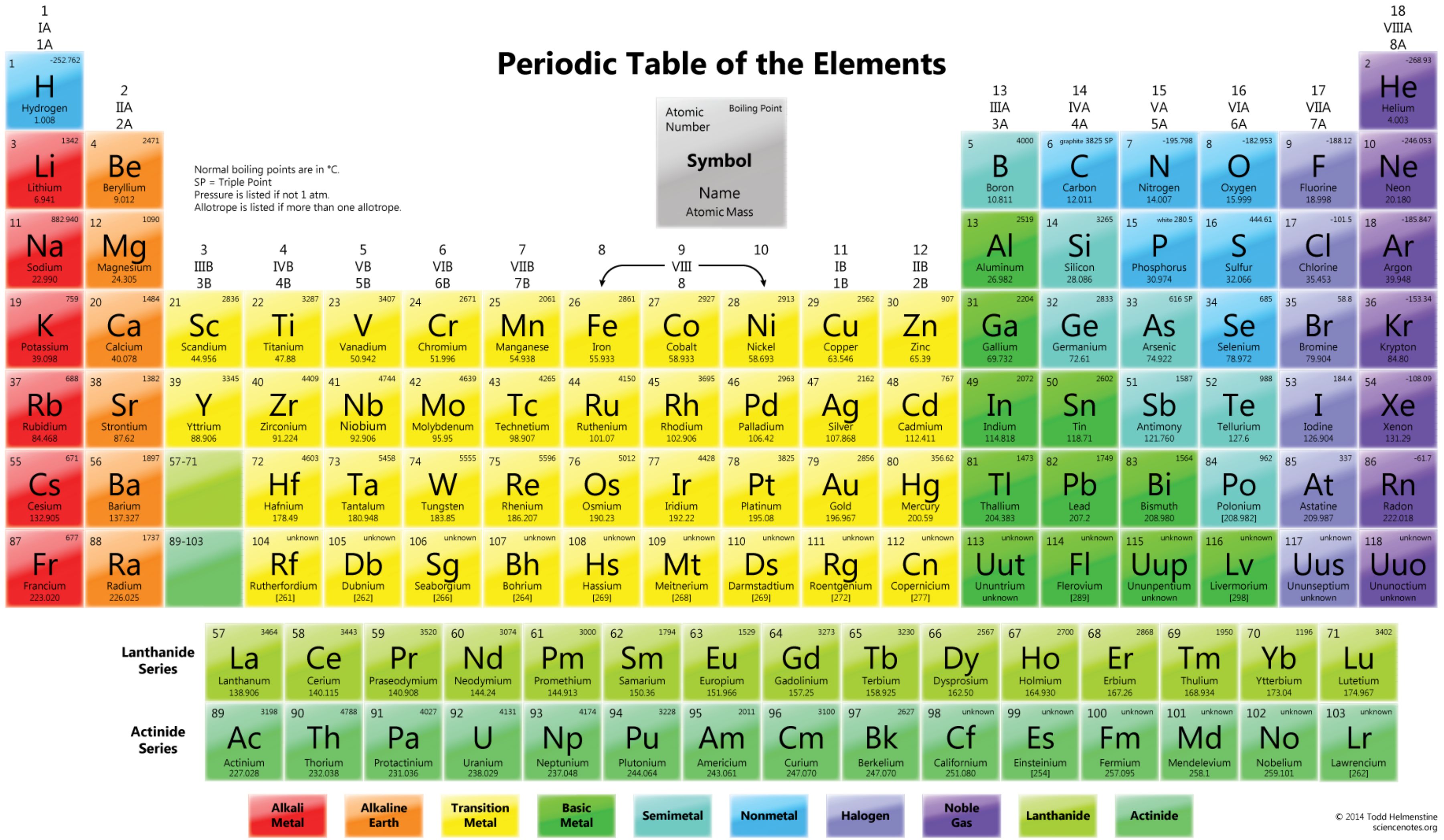
What Should We Build Our Chips Out Of?



Source: sciencenotes.org

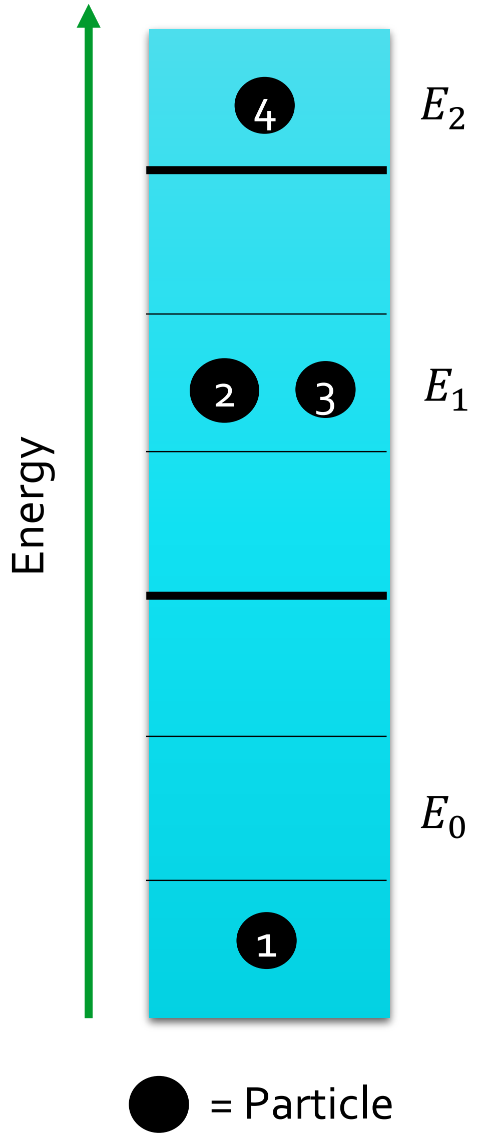
What is a Semiconductor?

» You've probably heard:
  - "It's a material that conducts better than an insulator, but not as well as a metal."

» This is true, but the full answer is much deeper

» Wasn't until the development of quantum mechanics in early 20th century that people understood why some materials conduct better than others

Quantum Statistics



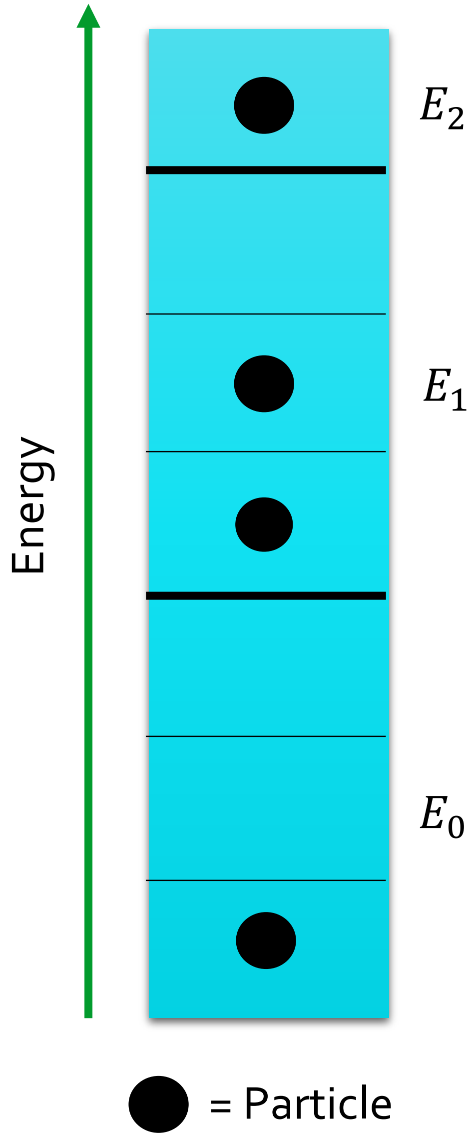
» What is the distribution of particles among the different energy states in a material?

» Boltzmann statistics
- Particles are distinguishable
- Multiple particles per state allowed

» Fermi-Dirac statistics
- Particles are indistinguishable
- Only one particle per state

» Bose-Einstein statistics
- Particles are indistinguishable
- Multiple particles per state allowed

Quantum Statistics



- Electrons and holes follow Fermi-Dirac statistics
- Number of ways to arrange the particles is

$$W = \frac{7!}{4!\,(7-4)!} = 35$$

- In general

$$W = \prod_{i=0}^{\infty} \frac{g_i!}{N_i!\,(g_i - N_i)!}$$

g_i: # of states at level i

N_i: # of particles at level i

Quantum Statistics

» What is the most probable distribution of particles?

$$\max \ln W \;\; \text{subject to} \sum_{i=0}^{\infty} N_i = N \, , \sum_{s=0}^{\infty} N_i E_s = E$$

» Solve using the method of Lagrange multipliers:

$$\max f(\mathbf{x}) \;\; \text{subject to } c_1(\mathbf{x}), = 0 \; c_2(\mathbf{x}), = 0 \;\; \ldots, c_m(\mathbf{x}) = 0$$

Vector of n unknowns

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) + \sum_{k=1}^{m} \lambda_k c_k(\mathbf{x})$$

Quantum Statistics

$$\nabla\mathcal{L} = \mathbf{0} \quad \rightarrow \quad \frac{\partial f}{\partial x_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial c_k}{\partial x_i} = 0, \forall\, i = 1,2, \dots n$$

$$c_k(\mathbf{x}) = 0, \forall\, k = 1,2, \dots m$$

- $n + m$ equations in $n + m$ unknowns
- For our problem:

$$f = \ln(W)$$

$$c_1 = \sum_{s=i}^{\infty} N_i - N \qquad c_2 = \sum_{s=i}^{\infty} N_i E_s - U$$

Quantum Statistics

» Our gradients become

$$\frac{\partial \ln W}{\partial N_j} \approx -\ln N_j + \ln\left(g_j - N_j\right)$$

$$\frac{\partial c_1}{\partial N_j} = 1, \qquad \frac{\partial c_2}{\partial N_j} = E_j$$

» Putting it all together:

$$\ln\left(\frac{g_j}{N_j} - 1\right) = -\alpha - \beta E_j \Rightarrow \frac{N_j}{g_j} = f\left(E_j\right) = \frac{1}{1 + e^{-\alpha - \beta E_j}}$$

α and β are our λ_1 and λ_2

Quantum Statistics

» Using the ideal gas law and the Maxwell-Boltzmann distribution, the multipliers α and β can be shown to be

Fermi energy ↓

$$\alpha = \frac{E_f}{kT} \qquad \beta = -\frac{1}{kT}$$

So,

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

Fermi-Dirac Distribution Function

Fermi Energy

- Note that E_f, by definition, is the energy level of the highest occupied energy level of the system at $T = 0$
- Equivalently, E_f is the energy level at which there is a 50/50 chance that a state is occupied

Band Theory of Solids

» Quantum mechanics is governed by the Schrodinger Equation, which describes the probability of particles being in particular energy states

» The 1-dimensional time-independent Schrodinger Equation:

$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0$$

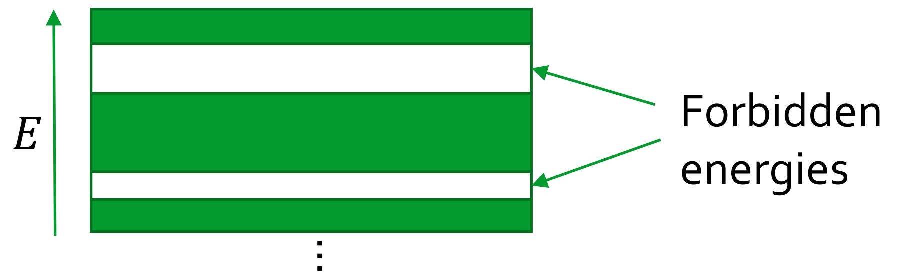
- ψ – Wave function
- m – Particle mass
- E – Total energy
- V – Potential energy

Band Theory of Solids in 1 Slide

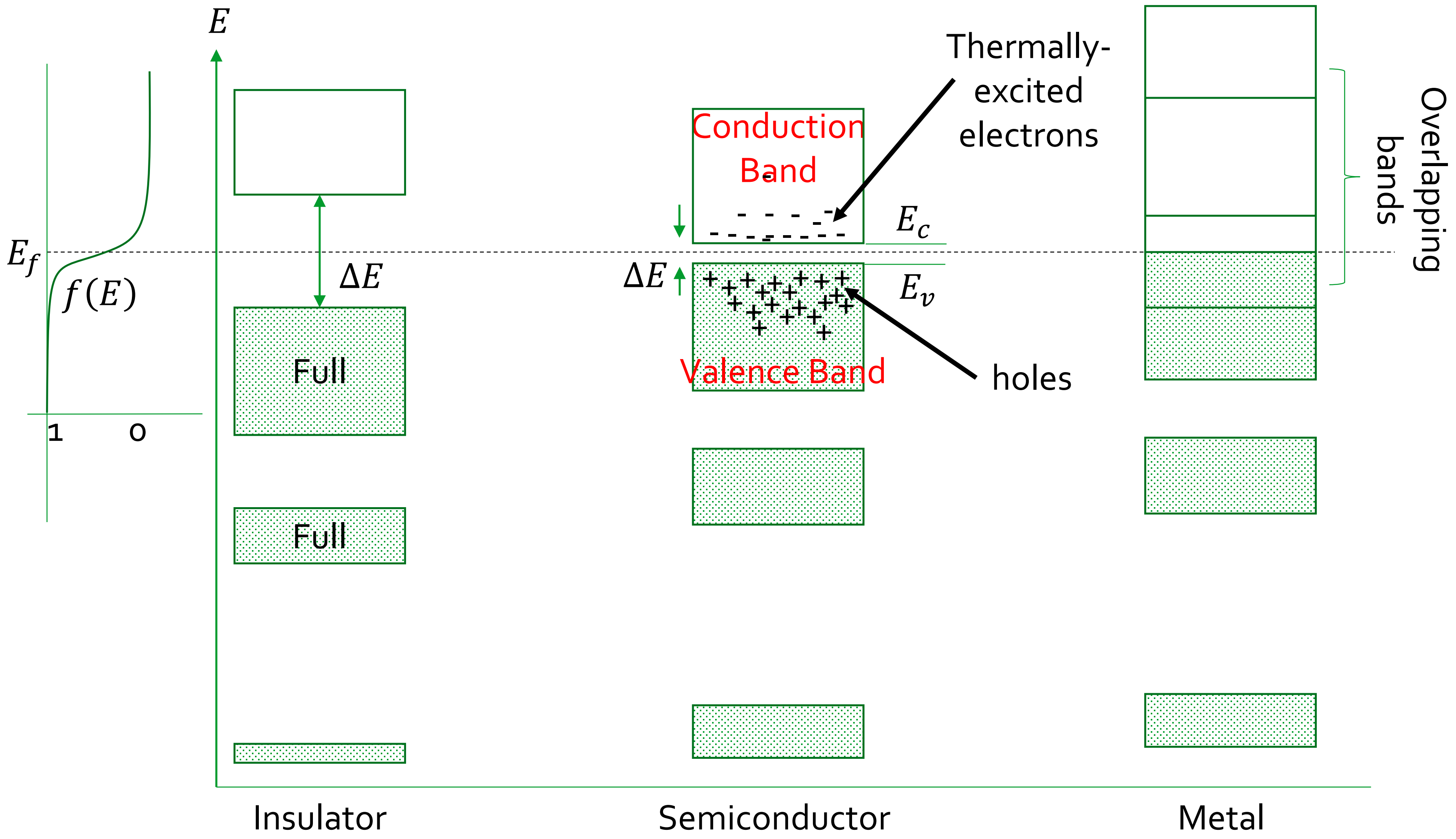
- Suppose $V(x)$ is a periodic function $u(x) = u(x + a)$
  - a is the period of u
- By the Bloch Theorem, the solution of the Schrodinger equation is:

$$\psi(x) = e^{ikx}u(x)$$

- In a crystalline sold, u is a square wave with peaks occurring at atomic nuclei
- Solving the Schrodinger equation using the Bloch Theorem with u equal to a square wave reveals bands of energy where no quantum states exist:



Insulators, Metals, Semiconductors



Adapted from J. McKelvey, *Solid State Physics*, Krieger, 1993.

Carrier Statistics of Semiconductors

» Two types of carriers:

- Electrons (n) – Excited into <u>conductance band</u> by thermal energy or applied electric field
- Holes (p) – Available electron state in <u>valence band</u> – "left behind" by excited electron

» Concentration of electrons in conduction band is

$$n_0 = \int_{E_c}^{\infty} f(E)g(E)\mathrm{d}E = c_n e^{-(E_c - E_f)/kT}$$

where c_n is a messy temperature-dependent term

» Similarly: $p_0 = c_p e^{-(E_f - E_v)/kT}$

» For *intrinsic* (pure) semiconductors: $n_0 = p_0 \equiv n_i$

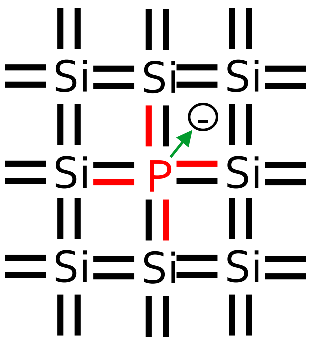
Extrinsic Semiconductors

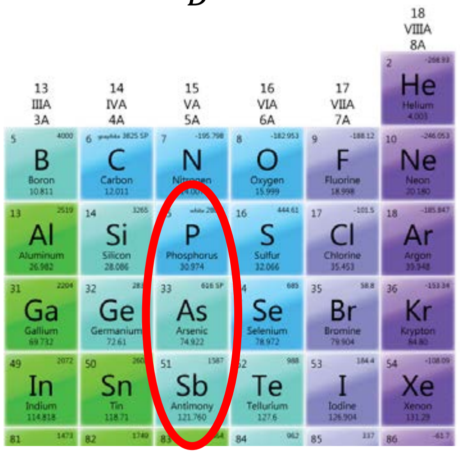
» Carrier concentrations of intrinsic semiconductors is too small to make good electronics

» Idea: Add dopants that can produce more carriers

n-type silicon:

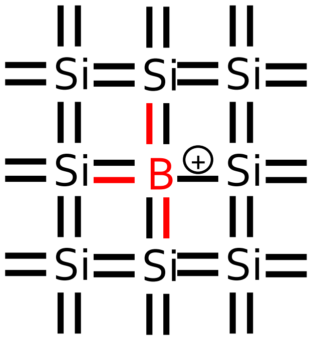
- Doped with donor atoms
- "Donate free electrons"
- $n_0 \approx N_D$
- $n_0 p_0 = n_i^2 \rightarrow p_0 \approx \frac{n_i^2}{N_D}$

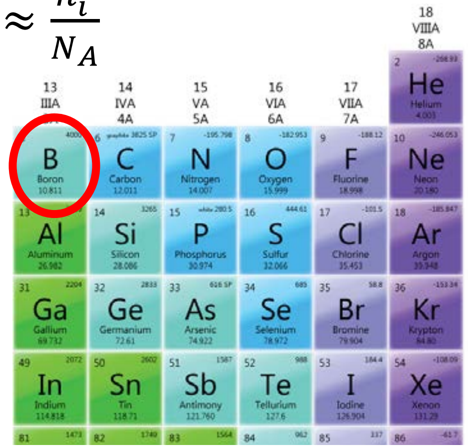




p-type silicon:

- Doped with acceptor atoms
- "Accept free electrons"
- Creates holes
- $p_0 \approx N_A$
- $n_0 p_0 = n_i^2 \rightarrow n_0 \approx \frac{n_i^2}{N_A}$





» Sometimes superscripts are used to denote relative dopant concentration (e.g. n^+ for highly-doped n-type, p^- for lightly-doped p-type, etc.)

So, Why Use Semiconductors?

- » Easy to manipulate properties through doping
- » Easy to control conductivity using electric fields
- » Why Si?
  - Forms good oxide (S_iO_2), which is critical to insulate/isolate devices and useful during fabrication process

Debeye Length